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A Suggestion for the Explanation of Stationary Radiant Points of Meteors. By H. H. Turner, M.A., F.R.S., Savilian Professor.

1. For many years past Mr. W. F. Denning has insisted that there are radiant points of meteors which remain fixed in the same portion of the sky for several months together. Indeed, he is disposed to believe that the Earth is liable to receive showers from the same radiant at all points of her annual orbit. It has been found difficult to explain this phenomenon theoretically. The conception of a swarm of meteors sufficiently extended to cross the Earth's orbit at many points is not difficult; but, if we assume them to be moving in parallel paths, the radiant, which depends on the relative motion of the swarm *and the Earth*, will shift among the stars during the year, as in the case of the well-known August *Perseids*. It would only remain approximately constant in position if the velocity of the meteors were enormously great compared with that of the Earth, a supposition which will not accord with the direct observations of velocity. Indeed, to quote the words* of Professor C. A. Young:—

"No satisfactory explanation of such fixity (of the radiant) as yet appears; and though Mr. Denning is perfectly confident of the genuineness of his discovery, and though it is very generally accepted as a fact, some very high authorities, Tisserand for instance, still question it, as being 'incredible and unaccountable.'"

2. My attention was drawn to this discrepancy between theory and observation more or less by accident. I know nothing of meteoric observation at first hand, but the confidence with which Mr. Denning speaks of the evidence in favour of stationary radiants seems to me sufficient to inspire others with similar confidence, and I devoted some little time to thinking over the matter. I was ultimately led to consider the effect of the Earth's action on that portion of the swarm which passed near, but did not meet, the Earth. Schiaparelli and others have fully considered the effect of the Earth's attraction in deflecting the paths of meteors which reach our atmosphere and become visible to us; but, so far as I have been able to ascertain, no one has yet considered fully the effect on the rest of the swarm. A brief note on the subject which I submitted to Professor Alexander Herschel received such favourable comment from him that I am encouraged to publish it. The original note was rather hurriedly drawn up, and is not reproduced verbatim, but is given in a revised form in the next five paragraphs.

3. Suppose, in the first instance, that a stream of meteors is

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* Young's *General Astronomy*, edition of 1898, p. 481.

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approaching the Earth at rest, and that there is no other disturbing body. Some of them, following paths C D E (fig. 1), will

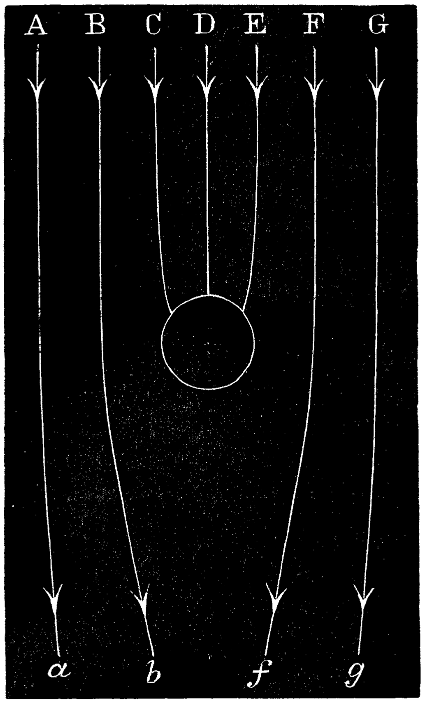

Fig. 1.

meet the Earth, or its atmosphere, and be stopped ; but others, further away, will follow paths such as A *a*, B *b* or F *f*, G *g*, which are hyperbolas of large eccentricity, if the original velocity of the meteors is such as occurs in nature.

4. The total effect of the Earth's attraction on any meteor path B *b* may be estimated by comparing the motions at two points, B and *b*, so far removed from the Earth on opposite sides that the Earth's attraction is very small. At these points the path is practically straight, and the velocity is the original velocity of the meteor stream. Between B and *b* the velocity gradually increases to a maximum at perigee, and then gradually decreases again to its old value, at the same time changing continually in direction. The change in direction will not be large if the original velocity of the meteors is such as is usually observed, *i.e.* comparable with that due to the Sun's attraction from infinity.

[For if V be the original velocity, and v the velocity at any point distant r from the Earth's centre, we have

$$v^2 = V^2 + \frac{2m}{r},$$

where m is the mass of the Earth.

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Also

$$V^2 \text{ is comparable with } \frac{2M}{R},$$

where M is the Sun's mass and R the distance of Earth from Sun. Thus the fractional increase in v^2 due to the Earth's action is of the order

$$\frac{2m}{r} \bigg/ \frac{2M}{R}, \text{ or } \frac{m}{M} \cdot \frac{R}{r}.$$

The smallest value of r is 4,000 miles, when the meteor just grazes the Earth's surface. Hence the greatest increment in v^2 is about

$$\frac{1}{330000} \times \frac{93000000}{4000} = \cdot 07 \text{ of the whole.]}$$

Nor will the change in magnitude be very great under the same circumstances. But it is to be noted that there is an accumulative effect on the *time* from B to *b*. At every intermediate point between B and *b* the velocity is greater than it would have been had the Earth not existed, and *the time between* B *and b is shortened* in consequence. It is this effect on which I wish to lay stress. It is the same for meteors on both sides of the Earth, such as B *b* and F *f*, whereas the change of direction is in opposite senses in these two cases. Thus if the same meteor could pass on one occasion to the left of the Earth, as along B *b*, and on a second occasion to the right, as along F *f*, the two changes of direction would annul each other, but the time-gains would add together.

Thus we may sum up the total action of the Earth as follows :—

(*a*) The velocity of a meteor after the encounter is unchanged *in magnitude*.

(*b*) The velocity after a single encounter is changed slightly in direction, but after two encounters in which the meteor passes on opposite sides of the Earth this change of direction may be annulled.

(*c*) But the *time of passing the Earth is shortened* : by which is meant the whole time spent between two points B and *b* at the limit of the Earth's sphere of influence.

5. Now consider the Earth in motion round the Sun as usual, and a meteor swarm about to cross its path. The swarm has

(A) the velocity of the Earth.

(B) the relative velocity of swarm to Earth.

Reverse for both bodies the velocity of the Earth, and during the subsequent motion reverse on both the effect of the Sun's acceleration on the Earth.

Then the motion of the swarm relatively to the Earth is given by supposing it to move with initial velocity (B) under accelerations due to

(C) the attraction of the Earth.

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(D) the *difference* of attractions of Sun on Earth and swarm, the familiar "disturbing force" of the Sun. Now, in Lunar Theory this disturbing force is shown to be small compared with the Earth's direct action at 240,000 miles distance. Thus at distances comparable with the Earth's radius of 4,000 miles it may be neglected entirely.

6. Hence for the study of the relative motion of swarm to Earth, we may regard the swarm as approaching a stationary Earth with the relative velocity (B) under (C) the Earth's attraction only. This is just the case considered in §3 and §4; and we may therefore apply the conclusions therein arrived at. In applying them it is to be remarked that there is no difficulty in imagining how the change of direction may be annulled as in (*b*) of § 4. For in the case of a swarm moving in an orbit the period of which is not commensurable with that of the Earth, any single meteor would pass at successive returns through the Earth's orbit in all positions relatively to the Earth indifferently—before or behind, near or far; and the average change of direction would be zero. Thus we may state the effect of the Earth's action on the motion of a meteor, on the average of a series of encounters in which the meteor escapes being actually stopped by the Earth, as follows:—

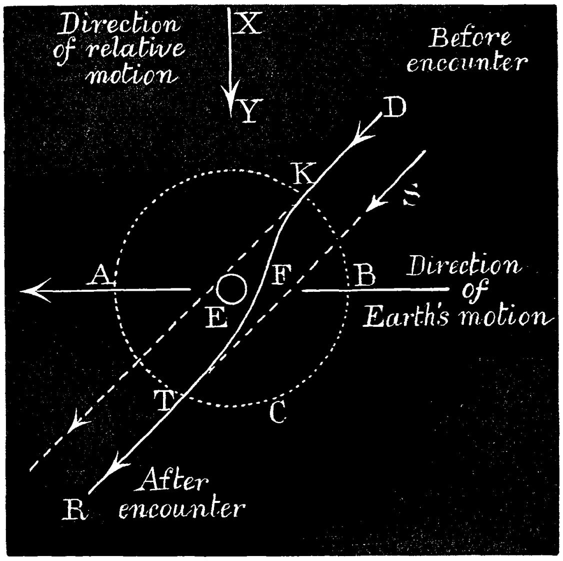

Fig. 2.

Average effect of Earth's attraction on motion of a meteor.

(*d*) The *relative velocity* is the same in *magnitude* after the encounters as before.

(*e*) The *relative velocity* is also the same in *direction* after the series of encounters as before.

(*f*) But on each occasion the meteors have crossed the Earth's

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orbit a little earlier than they would have done had the Earth not attracted them.

7. The average effect of the Earth's attraction on the motion in space is shown in fig. 2 (much exaggerated). Before the encounter the meteor's path is along D K ; and if the Earth had no attraction it would move with uniform velocity along the dotted line. It would appear to the Earth to come in a direction X Y, say.

Owing to the attraction of the Earth, which begins to be felt about K, the relative velocity in the direction X Y is increased, the orbital velocity common to Earth and meteor in the direction B A remaining the same. Hence the velocity of the meteor in space takes a direction more inclined to B A and it follows the curved path D K F T R, which only resumes its original direction at T R, when the increase of velocity has been destroyed and the Earth's attraction is again insensible. At the next return of the meteor it will arrive in the path S T R, which crosses the Earth's orbit a little sooner than the former; but it will approach the Earth with the same relative velocity both in *magnitude and direction* (on the average of several encounters) as at first. Since the position of the radiant depends simply on the relative velocity of meteor and Earth, the position of the radiant will be unaltered at the return, but meteors will be noted coming from it a little earlier than before. At the same time the Earth will only draw back in this way a few meteors of the swarm, while others will be left practically undisturbed. So that the Earth has a tendency to spread the meteor orbits along its orbit, the radiant remaining the same, but the time of the shower being gradually protracted. So far this action of the Earth seems promising as a *vera causa* for the existence of "stationary radiants."

8. But there are two important difficulties in extending this principle. The first is that if the action is slow (and the numerical computations which follow show how slow it is) it will have time to affect all the members of the swarm equally ; and thus, instead of a spreading of the swarm round the Earth's orbit, we shall merely have a progressive motion of the node for the whole swarm. To get a spreading action we must show cause why certain members of the swarm may be affected more than others ; and not only *more*, but *much more*, so that for some the node moves a considerable arc, while for others it stands still. If the meteors are all moving in precisely similar parallel paths it is not easy to see how this selective action is to take place. But is this a correct picture of the motion of a meteor swarm ? To keep the swarm extended must there not be some motion of rotation round an axis, as in the case of all other bodies of the solar system ? The total mass of the swarm being presumably small, a comparatively slight rotation would keep it extended, so that in considering the resultant velocity of any single meteor in space, due to its motion round the Sun and its rotation round the axis of the swarm, the latter may be neglected. But a rotation, however

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slight, would be important from our point of view for the following reason.

The axis of rotation might be inclined at any angle to the direction of motion; and the rotation could thus be resolved into components, one of which is about an axis coinciding with the direction of motion, the period being generally incommensurable with the time of orbital revolution. The effect of this component will be to transfer any particular meteor from one side of the centre to the other in an arbitrary manner; so that if the period of revolution of the swarm round the Sun were an exact number of years, the Earth's attraction might still cause no *deflection* of the path on the average. Thus suppose A B to be a portion of the Earth's orbit, C the position of the centre of the swarm when the Earth is at E. After an exact number of years, E returns to E and C to C; and if there were no rotation of the swarm, any particular meteor M would return to M. It would thus always pass the Earth on the same side, and not only the motion of the node would be multiplied at each return, but also the *deflection of the path.* But if there is any rotation, however slow, about the axis C E, then the meteor M may at the next return be at N, or any intermediate position; and so long as the time of rotation is incommensurable with that of revolution, the meteor will occupy in turn all positions in the circle described by M, half of which are to the right and half to the left of the Earth, so that the accumulated deflection due to the Earth's attraction is zero.

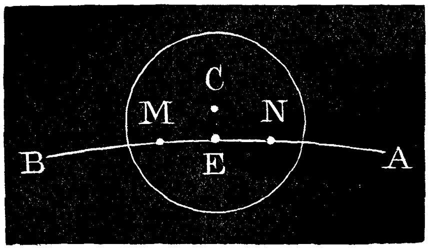

FIG. 3.

Thus, if the swarm rotates, we can have the phenomena of stationary radiation developing, even if the period of revolution of the swarm round the Sun is an exact number of years.

We can now explain the spreading of the swarm.

Let us suppose the original period $n+x$ years, where n is an integer and x an incommensurable fraction. The attraction of the Earth causes a motion of the node as above described: possibly common to the whole swarm—no scattering, all members of the swarm are in time equally affected. But the period slowly alters; since the *relative* velocity (to the Earth) remains the same, the velocity in space changes, and hence the period will change, either increasing or diminishing. It will thus gradually approach

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either n or $n+1$ years, *i.e.* it will approach commensurability. When the period becomes an exact number of years, some particular meteors will always pass very close to the Earth at every return, and the node of these will be moved rapidly. Others will be persistently avoided by the Earth, so to speak, and the node will remain unchanged. Hence the swarm will be split up. Again, for the portion which moves onward the period will gradually become incommensurable, and thus the action slower; but it will nevertheless proceed, and will carry the node round, further altering the period until it is again commensurable (in $n-1$ or $n+2$ years), when a portion of this portion will again be detached, and so on. The disintegrating action thus takes place more or less in steps, though there is no actual discontinuity. Of course, there are minor steps when x becomes not ± 1, ± 2, ± 3, &c., but $\pm\frac{1}{2}$, $\pm\frac{1}{3}$, &c.

9. The second difficulty is more important. According to the principle under consideration the relative velocity of the meteors to the Earth remains unchanged, not only in direction (on the average), but also in magnitude. Now, I gather that this does not accord with observation. Meteors which *meet* the Earth in its orbit are observed to be swift, and those which *catch it up* are slow; and this, I am told, is the case for meteors belonging to the same stationary radiant. We have thus proved too much, if anything. We want the relative velocity to remain constant in direction, but *not in magnitude*. This is a formidable obstacle to the application of the above principle, and at present I do not see the way to surmount it. For diminishing the magnitude of a velocity without altering its direction, a resisting medium seems a likely agent; and if there is a resisting medium, small bodies like meteors should be specially susceptible to its influence. Now, if we had only the meteors near the apex and anti-apex to consider, an explanation of the swift velocities near the apex, and the slow near the anti-apex, might be offered as follows:—

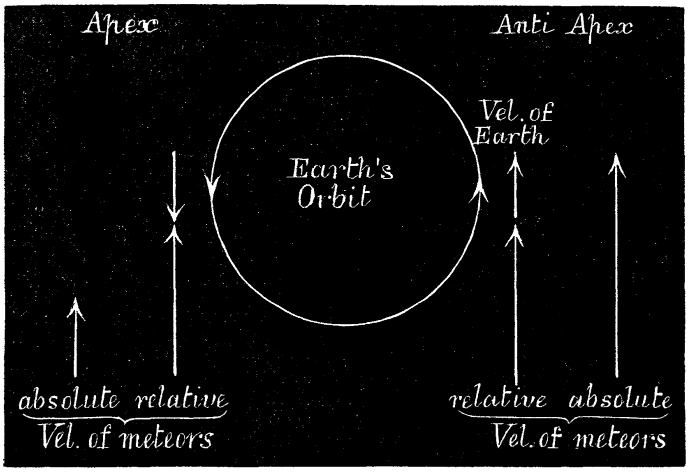

Fig. 4.

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Suppose, first, that the meteors had been scattered round the Earth's orbit as above indicated, the relative velocity being constant both in direction *and magnitude.* Then at the apex the velocity of the meteors in space is the difference of the relative velocity and that of the Earth, and is small; at the anti-apex it is the sum of these velocities, and is large. Hence the latter would be reduced by a resisting medium fixed in space much more than the former: so that the relative velocity would become smaller at the anti-apex than at the apex.

But at intermediate points the relative velocity would be changed not only in magnitude, but in direction, by such a diminution of the absolute velocity. This explanation breaks down.

10. In spite of this difficulty, I venture to publish this suggestion, in the hope that it may perhaps at least draw attention to the important problem of stationary radiants. I am well aware that some experienced meteor-observers deny their existence altogether, alleging, for instance, that Mr. Denning has obtained his results by confusing together meteors belonging to different swarms, straining observations, &c. My own opinion is that this interpretation of Mr. Denning's observations is impossible, and that we are face to face with another of those cases in which observation supplies us with facts apparently inexplicable by theory at present, for which a theoretical explanation will yet be found. It is therefore a question merely of attracting sufficient attention to the matter; and the object of this note will have been attained if it succeeds in attracting a little more attention from mathematicians to the present discrepancy between observation and theory. In this course I am glad to think that I have the approval of Mr. Denning, and of Prof. A. S. Herschel, who has kindly added in commentary a paper of his own.

11. The foregoing sketch of the Earth's action would be incomplete without some rough numerical estimate of its amount, which will now be attempted.

12. Let us first examine for what distance a meteor may be regarded as travelling under the Earth's attraction only. This will depend on the velocity of approach. Meteors which are seen in our atmosphere have parabolic or nearly parabolic velocities: that is, the periods of revolution round the Sun must be several years at least. We may, of course, disregard those with velocities greater than parabolic: for they will not return, and we are considering periodic swarms. Thus we may consider the maximum velocity of a meteor in space when it meets the Earth's orbit to be V, where

$$V^2 = 2M/R,$$

M being the Sun's mass and R the radius of the Earth's orbit. Since it may approach the Earth at the apex, its maximum *relative* velocity is V + W, where W is the velocity of the Earth, *i.e.*

$$W^2 = M/R.$$

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Thus

$$(V+W)^2=(\sqrt{2}+1)^2 M/R$$
$$=5\cdot 8\ M/R;$$

while at the apex the relative velocity would be V − W, where

$$(V-W)^2=(\sqrt{2}-1)^2 M/R$$
$$=\cdot 17 M/R.$$

As a typical case we shall consider the relative velocity to be V; the other cases can easily be considered afterwards.

Let r be the distance of the meteor from the Earth, mass m. The "disturbing force" of the Sun, *i.e.* the difference between attractions of Sun on meteors and Earth is of the order

$$M.r/R^3,$$

while the direct attraction of the Earth is of the order m/r^2. These two become equal when

$$(r/R)^3=m/M=1/330{,}000,$$

or

$$r=R/70=1{,}300{,}000 \text{ miles}.$$

At half this distance the disturbing force (which varies as r^3 compared with Earth's action) has only $\frac{1}{8}$ the effect. Hence we may consider the meteors as moving under Earth's attraction only for about 500,000 miles on each side of the Earth's orbit, but not more.

With velocity V, which is 1·4 times the Earth's velocity of 1,500,000 miles per day, they would take about 12 hours to describe this double-space. With velocity V + W it would be about 3 hours; with velocity V − W, about 3 days.

13. In § 4 it is shown that the increase of velocity due to the Earth's action is very small. Let us further calculate the minimum eccentricity of the hyperbola described relatively to the Earth. If a be the major axis, e the eccentricity, v the velocity at distance r from the Earth,

$$v^2=m\left(\frac{2}{r}+\frac{1}{a}\right);$$

and when r is infinite

$$v^2=V^2=2M/R.$$

Thus

$$m/a=2M/R,$$

or

$$a=141 \text{ miles}.$$

Now the minimum perigee distance is 4,000 miles; thus

$$a\ (e-1)>4000$$
$$\therefore\ e-1>4000/141,$$

or

$$e>30.$$

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If instead of V we took V − W, we should obtain

$$a = 1{,}700 \text{ miles},$$
$$e > 3{\cdot}3.$$

The perigee distance will, of course, always exceed 4,000 miles, so that e and also $(e-1)$ will generally be large quantities, and we may write $e-1=1/f$ where f is small, and only first powers of f need be retained.

14. To calculate the total time-gain due to the Earth's attraction, let us suppose that the path of a meteor is *constrained to be a straight line*, which makes the integration a little simpler, while clearly not seriously affecting the result.

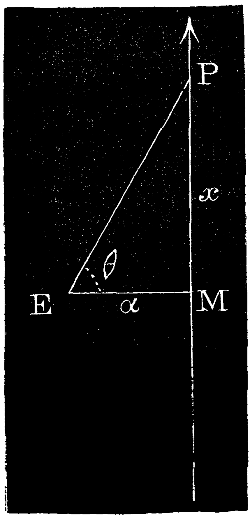

Fig. 5.

Let a be the perigee distance E M (fig. 5), and let M P $= x$, M E P $= \theta$, where P is the position of the meteor at time t; so that

$$x = a \tan \theta.$$

The equation of motion is

$$\frac{d^2x}{dt^2} = -\frac{m}{x^2}\sin\theta,$$

or

$$\frac{dx}{dt}\cdot\frac{d^2x}{dt^2} = -\frac{m}{a}\sin\theta\,.\,d\theta$$

$$\therefore \left(\frac{dx}{dt}\right)^2 = \text{C} + \frac{2m}{a}\,.\cos\theta.$$

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When $\theta = \frac{\pi}{2}$,

$$\left(\frac{d\,x}{dt}\right) = V^2 = \frac{m}{a} = \frac{m}{a}\ (e - 1) = \frac{m}{af},$$

$$\therefore\ \ a \sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt} = V\ (1 + 2\ f \cos\ \theta)^{\frac{1}{2}},$$

$$\therefore\ \ V\ dt = a\ \sec^2\theta\ .\ (1 - f \cos\ \theta) d\theta,$$

or

$$Vt = a\ \left[\tan\ \theta - f \log_e \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right],$$

no constant being required if θ and t vanish together.

Now if the portion of the path under consideration commence and end at a distance of 500,000 miles from the Earth, perigee distance being 4,000 miles, the values of tan θ at the limits are ± 125, and of θ are $\pm 89^\circ\ 33'$

$$\log_e\ \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \pm\ 5{\cdot}5$$

Thus in

$$t = \frac{a}{V}\left[\tan\ \theta - f\ \log_e\ \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right],$$

where the first term clearly corresponds to the uniform (undisturbed) motion in a straight line, and the second to the effect of the Earth's attraction, the ratio of the second term to the first is

$$f \times 5{\cdot}5/125\,;$$

or if $f = 1/30$, the ratio is $1/700$ say. Thus, if the undisturbed time be twelve hours (see § 12) the time-gain is about *one minute*. Near the apex, where the relative velocity is $V - W$, f is much larger and also the time within which we integrate is larger; and thus the time-gain would for both reasons be larger. The time-gain in the case where the meteor just misses the Earth is in fact represented approximately by

$$f\,.\,T/23,$$

where T is the time spent within the sphere of influence of the Earth. Thus at the apex, where $T = 3$ days (see § 12) and $f = 1/3{\cdot}3$, we have for the time-gain about one hour instead of one minute; at the anti-apex, on the other hand, the minute becomes a few seconds only.

15. But so long as the effect is shown to be sensible, it is not a fatal objection that its action would take a long time to develop. There is no reason that I know of why the swarm should not have been subjected to the Earth's attraction at successive returns for, say, 1,000,000 years. If n years be the period of commensurability (as suggested in § 8) for some portion of the swarm, and this portion be moved one minute at each return,

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then in 525,600 n years the node for this portion would be moved completely round the Earth's orbit. For a smaller spreading smaller figures would be necessary.

16. In such an important matter it is of course eminently desirable to undertake a more elaborate investigation, and this I shall hope to do. Meanwhile these notes may at least, as suggested in § 10, have the desired effect of attracting attention to an interesting problem.

Observations of the Leonids at Perth Observatory, Western Australia. By W. E. Cooke.

A search was made for these at the Perth Observatory, W.A., on November 13 and 14. We confined our observations to counting the number seen in a particular region of the sky during each successive 15 minutes.

On the first night I took a circle of 10° radius round *Aldebaran*, and Mr. Yeates took the same-sized region round *ζ Leonis*. The night was brilliantly clear.

On the 14th Mr. Yeates took the same region as before. Mr. Curlewis took *Albebaran*, and I took an irregularly shaped region visible through the shutter opening of the Dome, between *Sirius* and *Procyon*. My observations were slightly interrupted at times, as I had strapped a small camera on the astrographic mounting, and had to attend to this occasionally. The sky was generally clear, but some very thin stratus cloud drifted across from time to time.

The photographs produced no results of any value.

Results.

1898, November 13.

Observer	Region round	From 6.15 to 6.30	6.30 6.45	6.45 7.0	7.0 7.15	7.15 7.30	7.30 7.45	7.45 G.M.T. 8.0 ,,
Cooke	Aldebaran	...	...	...	0	1	0	0
Yeates	ζ Leonis	...	...	..	1	0	2	1

November 14.

Cooke	Sirius to Procyon	0	0	0	0	0	0	0
Curlewis	Aldebaran	0	2	1	1*	2	2†	3
Yeates	ζ Leonis	1	2	1	5	0	3	0

Perth Observatory, W.A.
1898, *November* 16.

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* A brilliant one, straight through *Aldebaran* from the direction of *ζ Tauri*.

† A very brilliant one, just below *Aldebaran*, parallel to above, leaving luminous trail after disappearance.